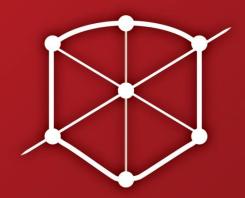
#### AI\*IA Awards: In Memory of Leo, Lesmo Award for Best Thesis



Surveilling and protecting valuable targets exploiting a spatially uncertain alarm system

Giuseppe De Nittis

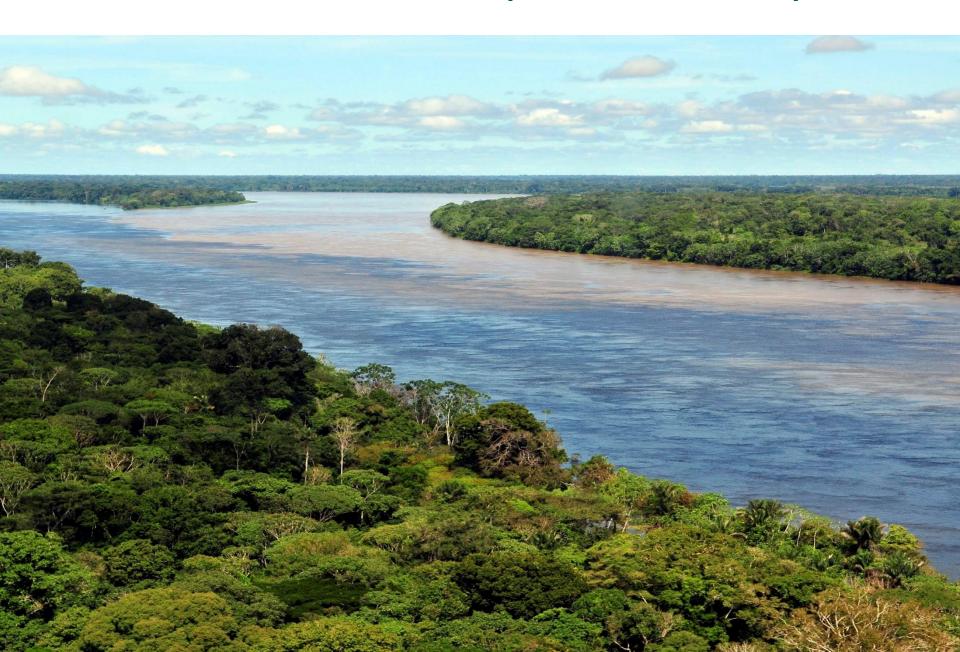
Supervisor: Nicola Gatti

Co-Supervisor: Nicola Basilico





# Anavilhanas natural reserve (about 4000 Km<sup>2</sup>)



# Flying drone



# A spatially uncertain signal



# One signal, multiple targets



# **Security games**



The Defender controls resources to protect the environment

The Attacker tries to compromise some areas without being detected



# **History**



Los Angeles, 2008 AAAI, AAMAS



U.S. domestic flights, 2009 AAMAS



Milan, 2015

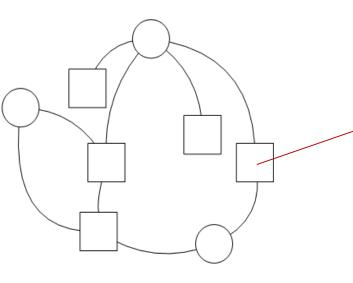


#### The model









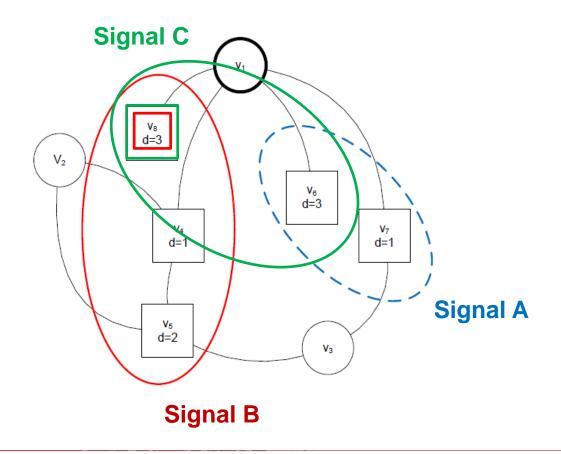
#### Target t:

- $\pi(t)$ : value
- d(t): penetration time



### The alarm system

When a target is attacked, a spatially uncertain signal is generated





#### The actions

#### At any stage of the game:





The Defender decides where to go next

The Attacker decides whether to attack a target or to wait



#### **Utilities**





$$U({c, t}) = (1, 0)$$



$$U({c, t}) = (1 - \pi(t), \pi(t))$$



# Solving the game







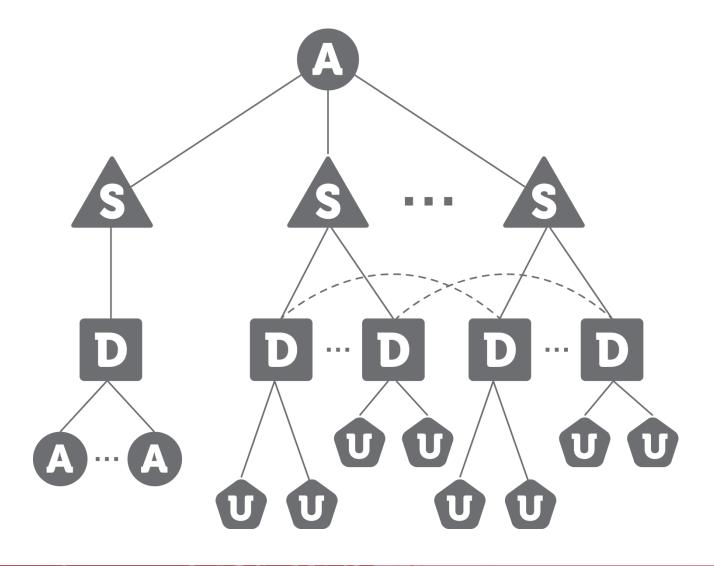
The Defender's strategy is common knowledge of the game



We adopt a Stackelberg paradigm, reducing to a maxmin equilibrium

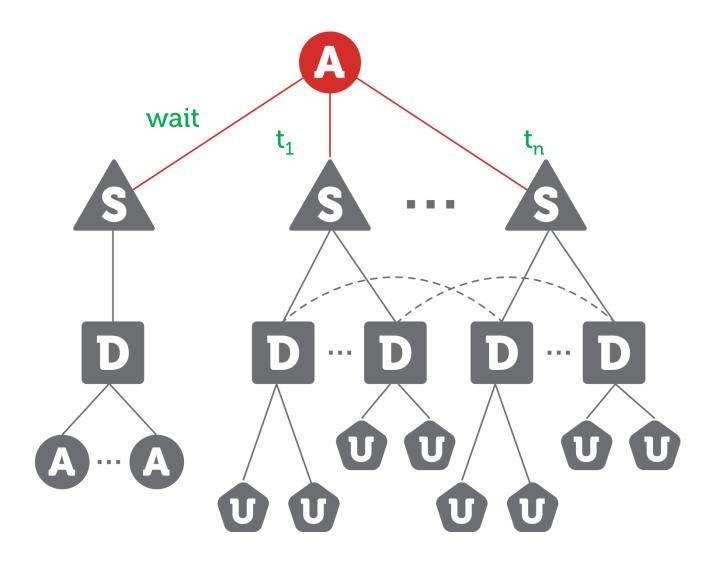


### **Interactions**



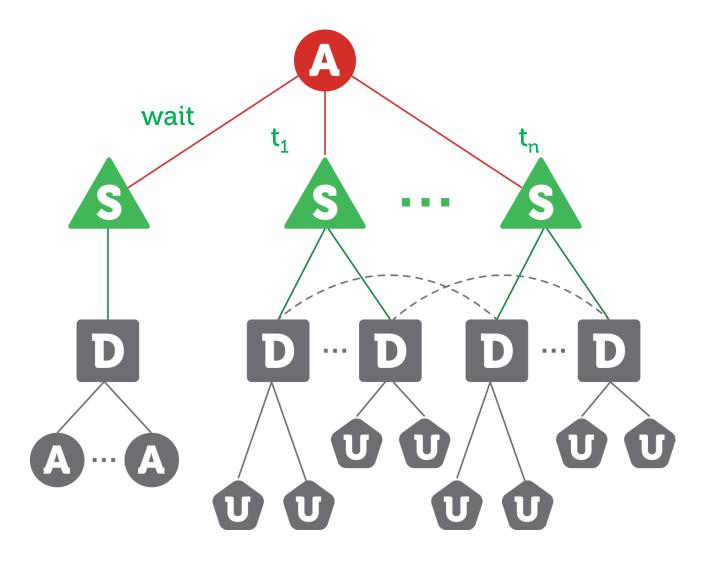


#### The attacker's action



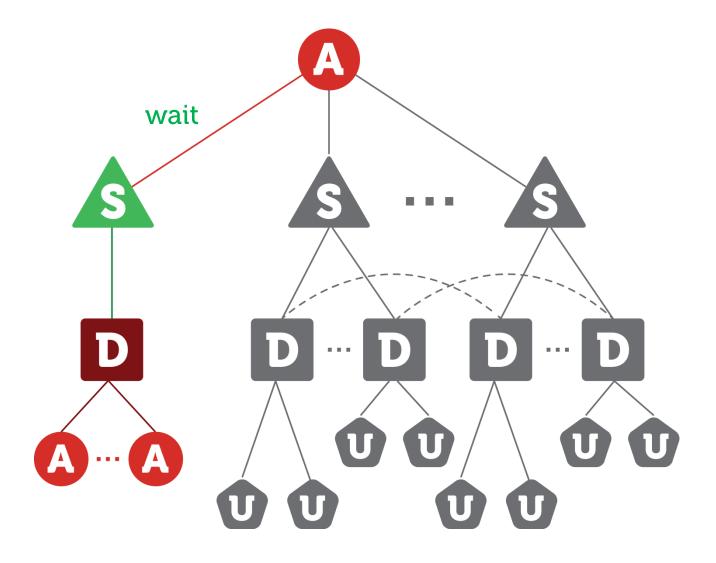


# The alarm system



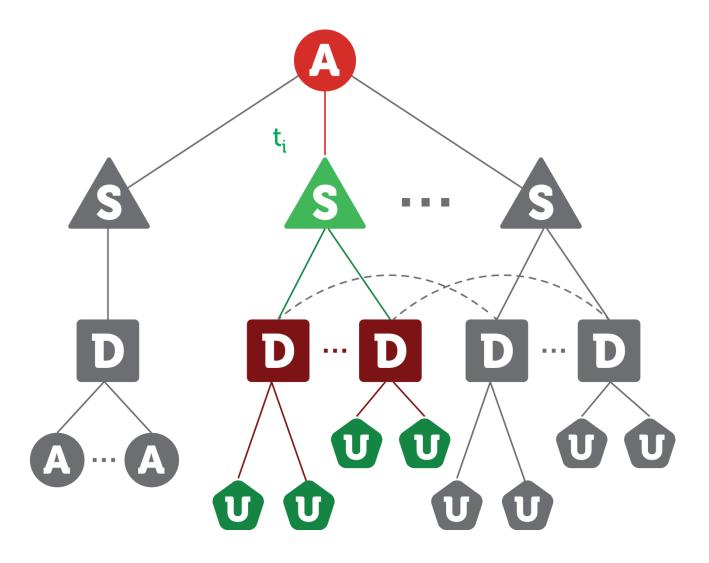


# Patrolling Game (PG)



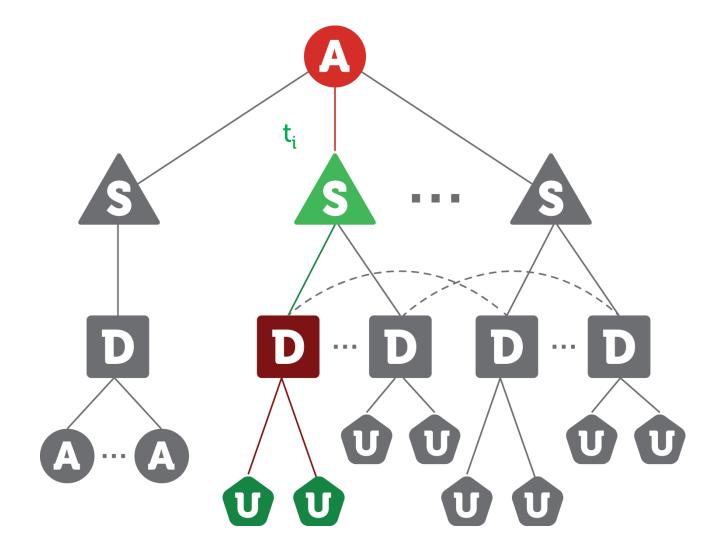


# Signal Response Game (SRG)



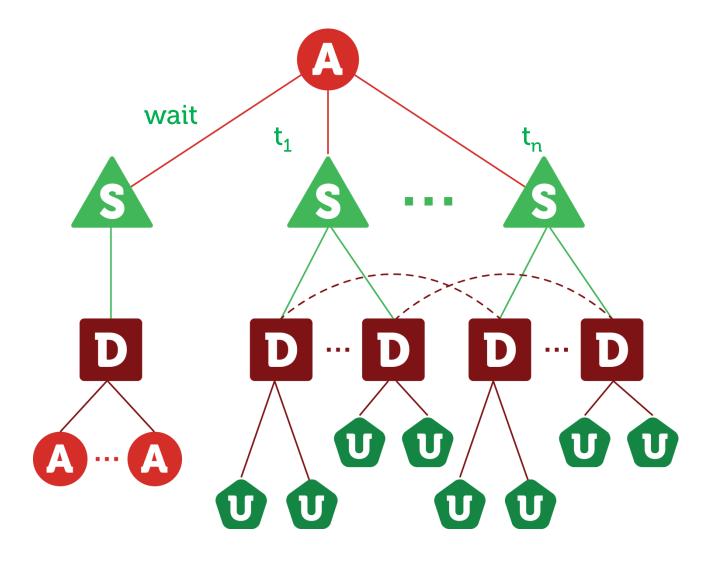


# SRG-v





### **Interactions**





# Two phases of the game





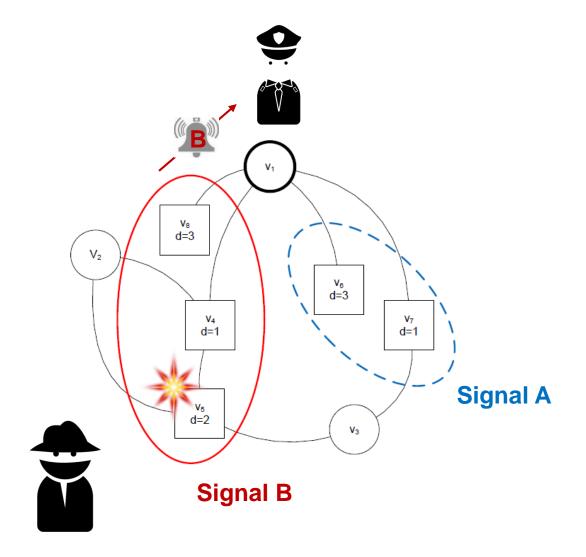


Normal patrolling

Signal response

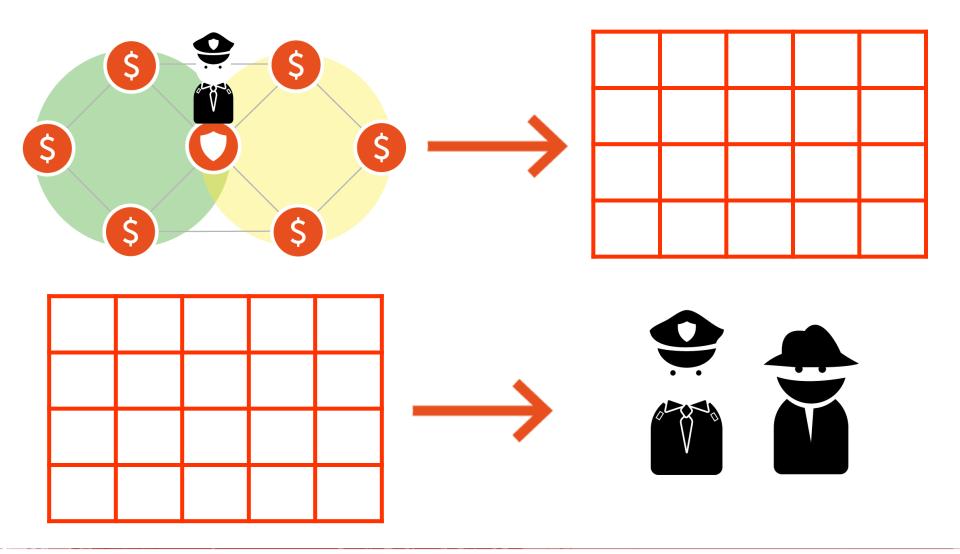


### The SRG-v





# The two halves of the SRG-v problem



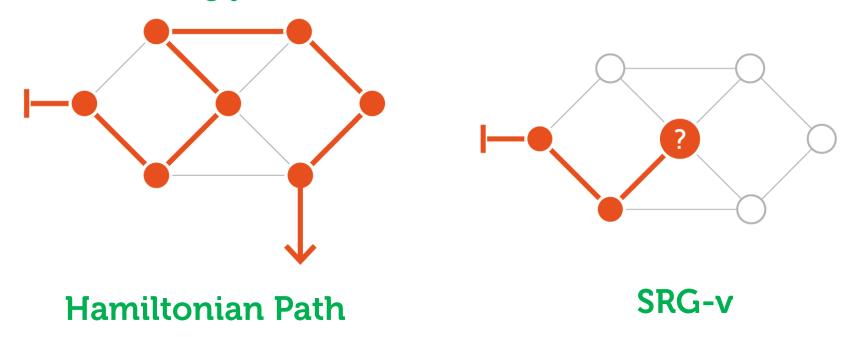


### A hard task: SRG-v on arbitrary graphs

**INSTANCE**: an instance of SRG-v

QUESTION: is there any  $\sigma^D$  such that  $g_v \le k$ ?

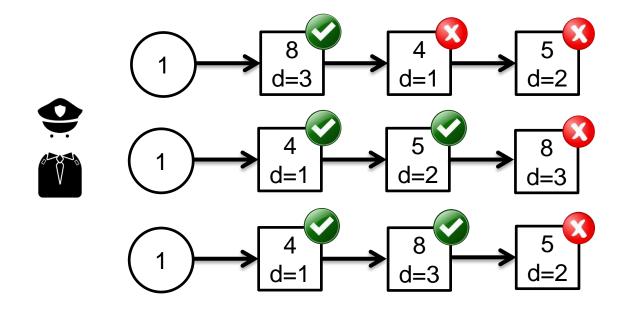
k-SRG-v is strongly NP-hard even with |S| = 1.

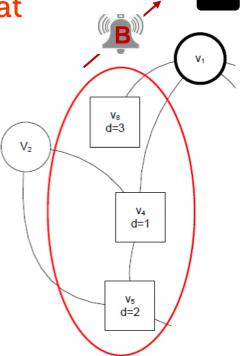




## **Covering route**

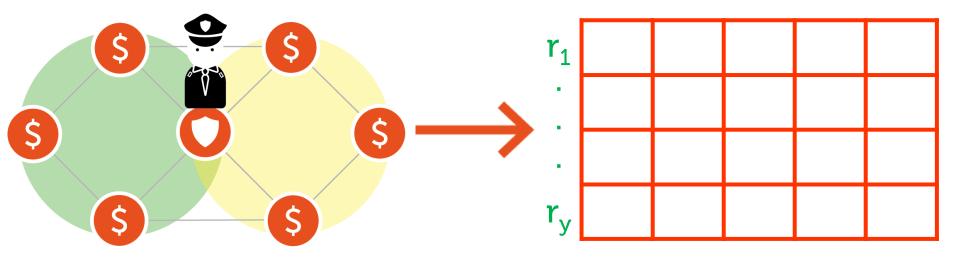
A permutation of targets that specifies the order of first visits (covering shortest paths) such that each target is first-visited before its deadline







# **Building the game**



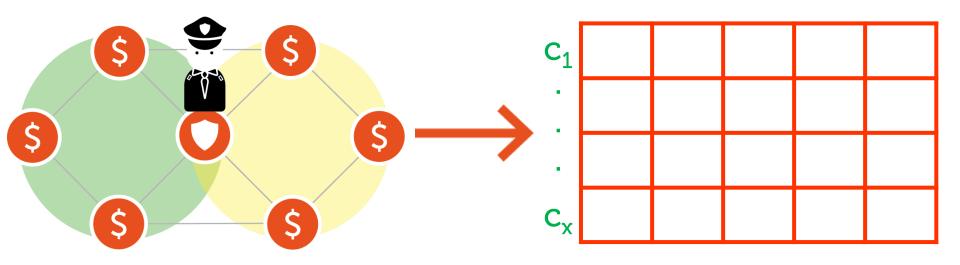
Complexity: O(n<sup>n</sup>)



# **Covering sets**

Can we consider covering *sets*?

From  $\langle t_1, t_2, t_3 \rangle$  to  $\{t_1, t_2, t_3\}$ 

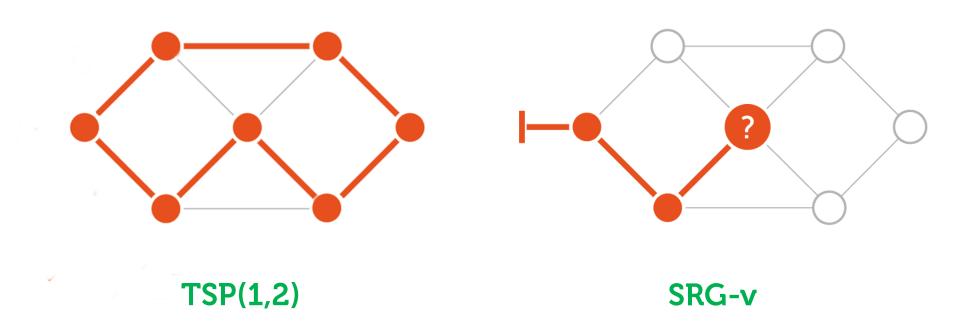


Complexity: O(2<sup>n</sup>)



### Approximating SRG-v game value

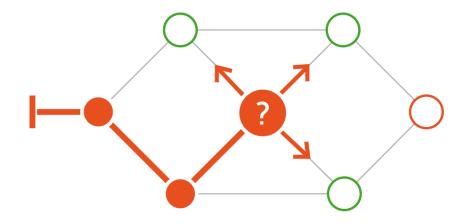
The optimization version of k-SRG-v is APX-hard even for very simple instances





# Our algorithm

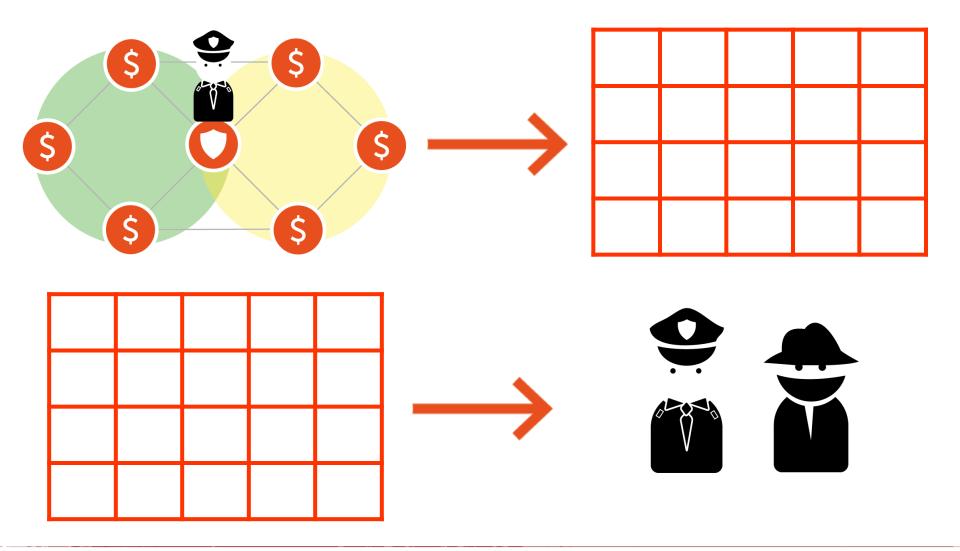
We simultaneously build covering sets and the shortest associated covering route



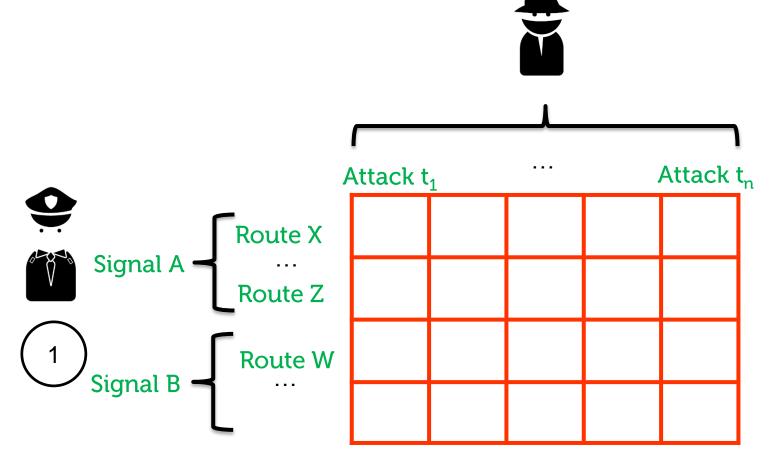
Dynamic programming inspired algorithm: we can compute all the covering routes in  $O(2^n)$ 



## A hard task



#### Solution of the SRG-v



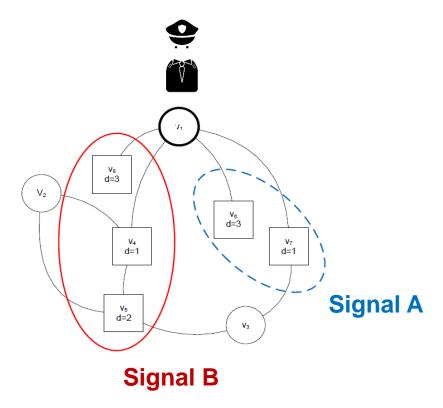
Route Y

Complexity: O(nc)



### The Patrolling Game

What to do when no signal is received?

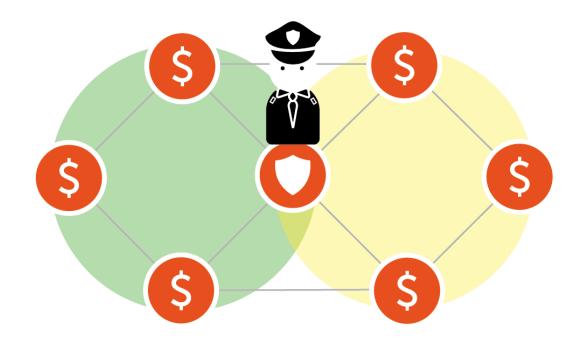


The Attacker can observe the position of the Defender



#### Stand still

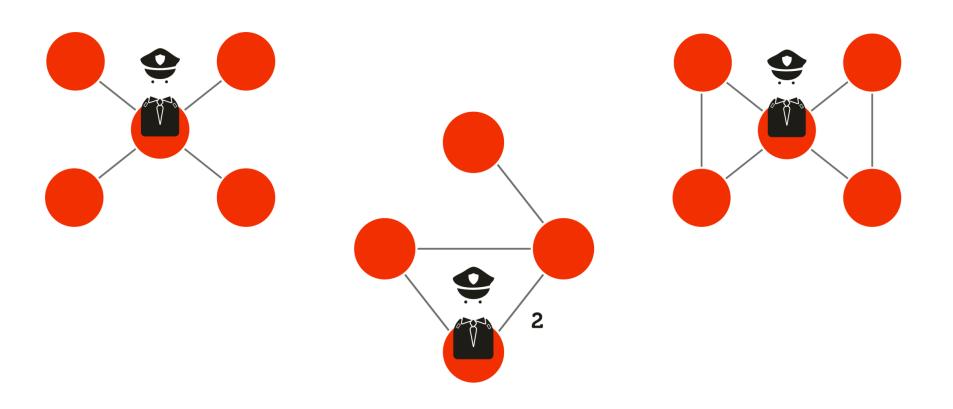
Without false positives and missed detections, if the alarm system covers all the targets, then any patrolling strategy is dominated by the placement in v\*





### Special instances

There exist Patrolling Games where staying in a vertex, waiting for a signal, and responding to it is the optimal patrolling strategy for D even with a missed detection rate  $\alpha$  = 0.5





## **Experimental campaign**

Hard instances: up to 20 targets

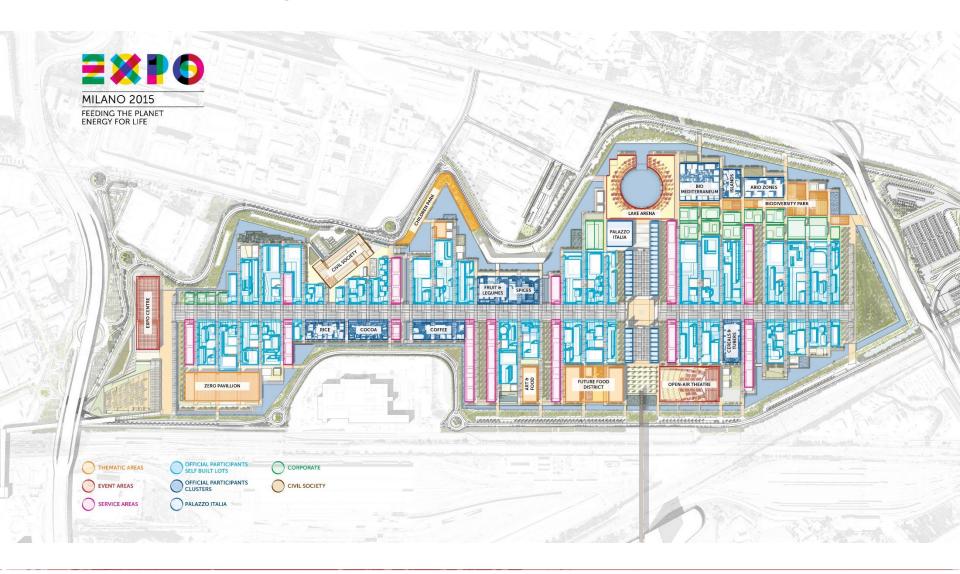
Require the computation of an Hamiltonian Path

Normal instances: up to 200 targets

- Low edge density
- Spatial locality: distant targets covered by different signals

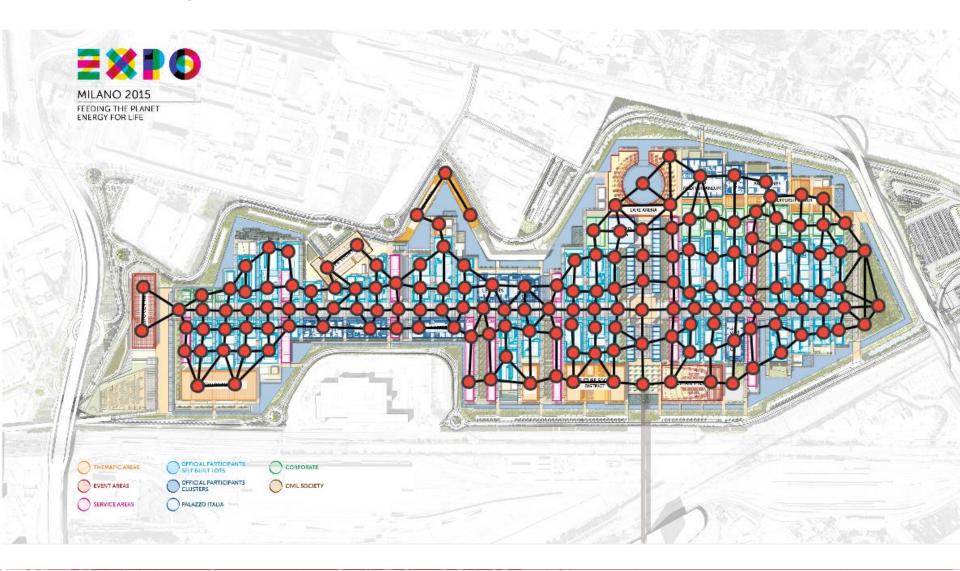


# Expo: the setting



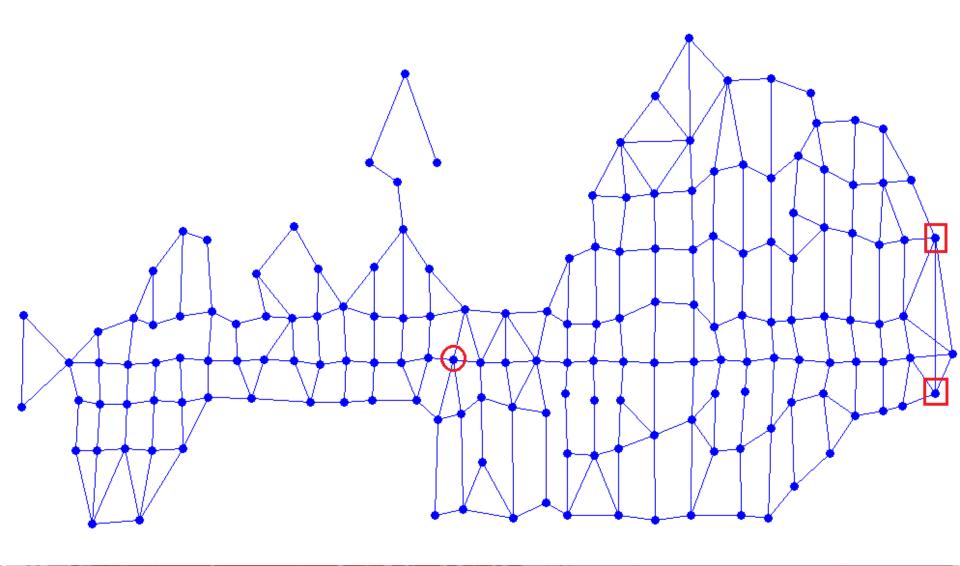


# Expo: the graph



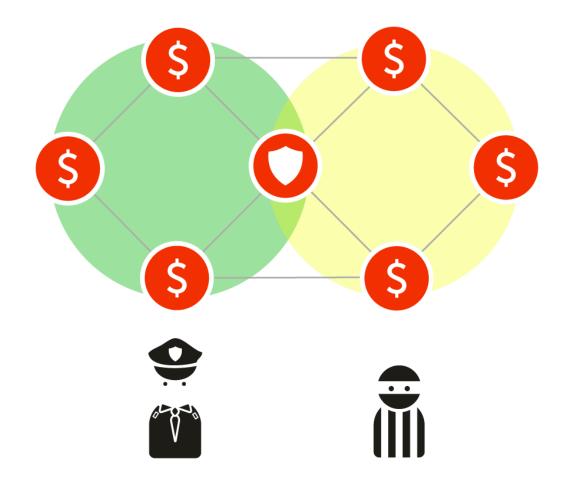


## Expo: the solution



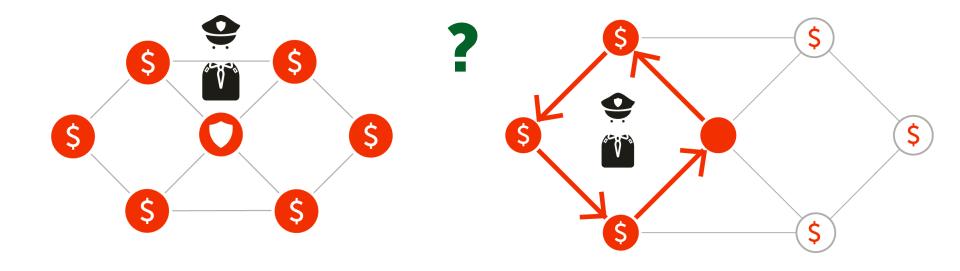


#### **Conclusions**

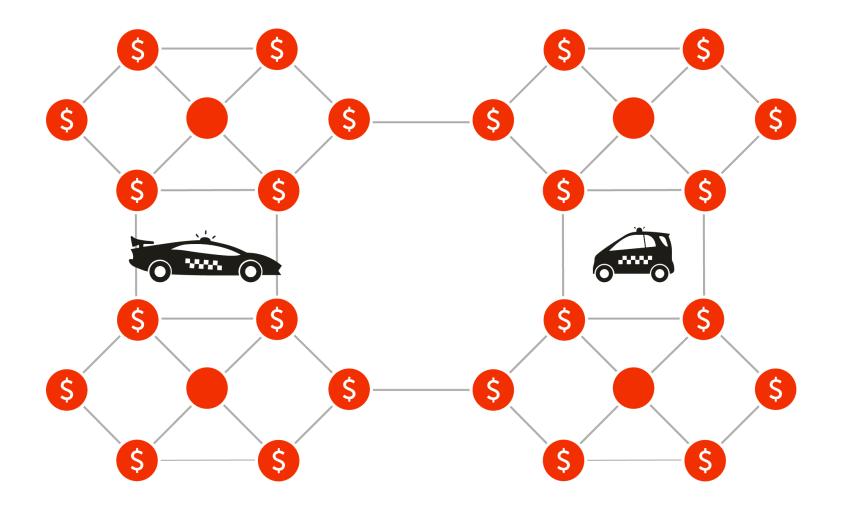




#### **Current research: missed detections**



## Future research: multi-patrolling





### Future research: alarm system deployment







#### **Our Team**



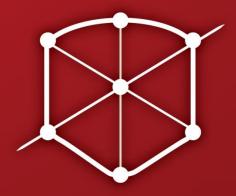
Nicola Gatti



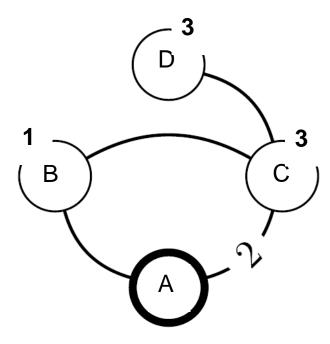
Giuseppe De Nittis



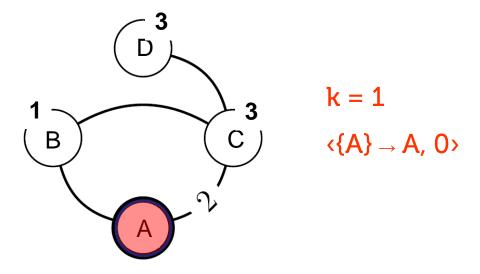
Nicola Basilico



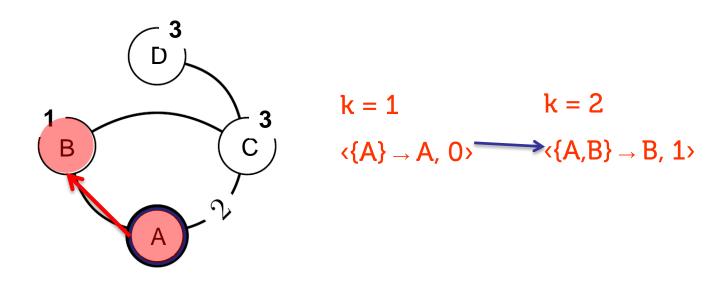
Thank you!



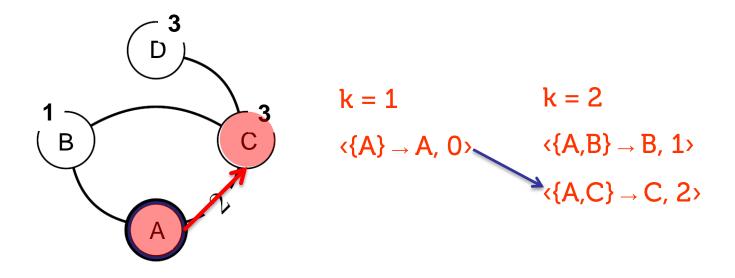




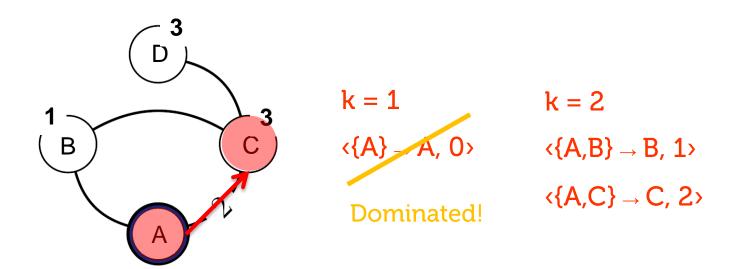


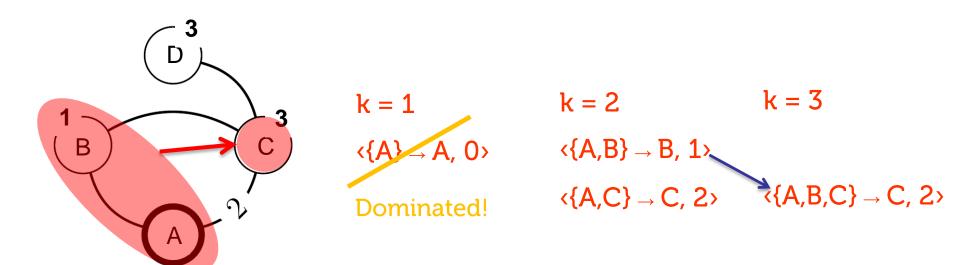


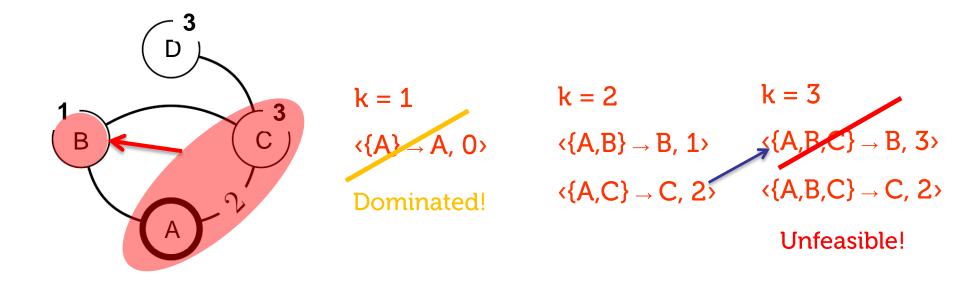
### Our Algorithm: an Example

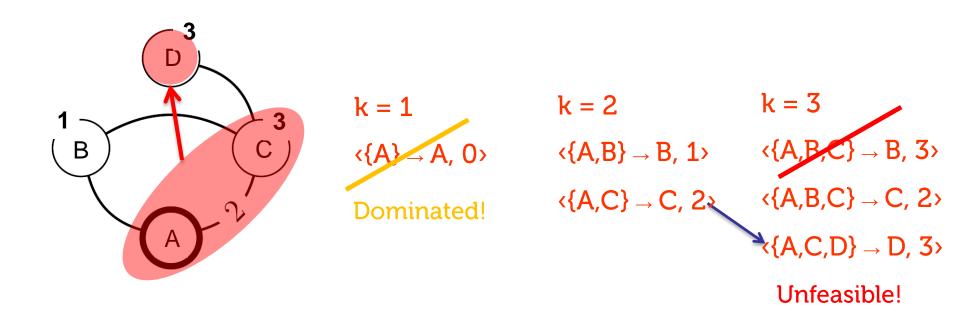




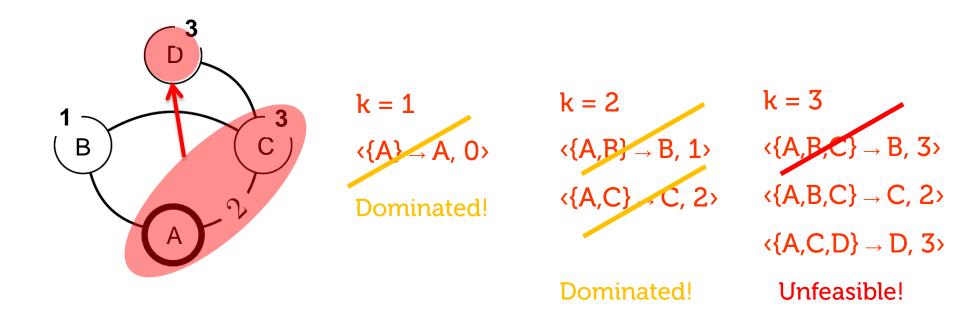


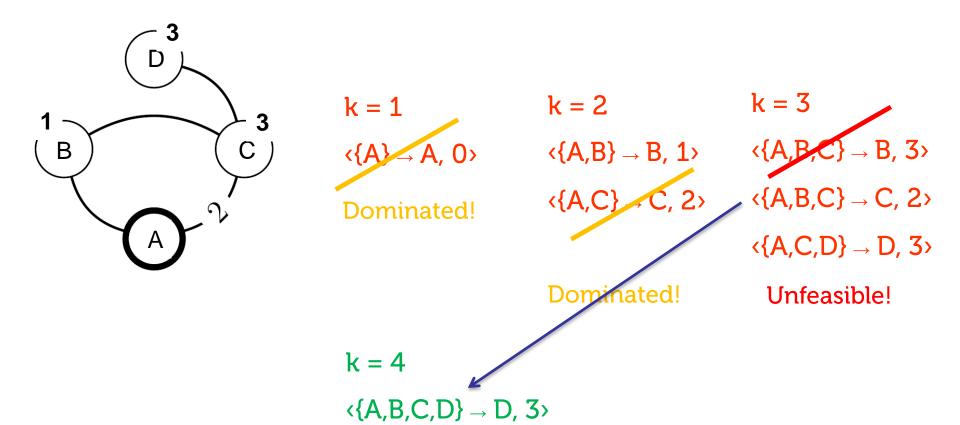




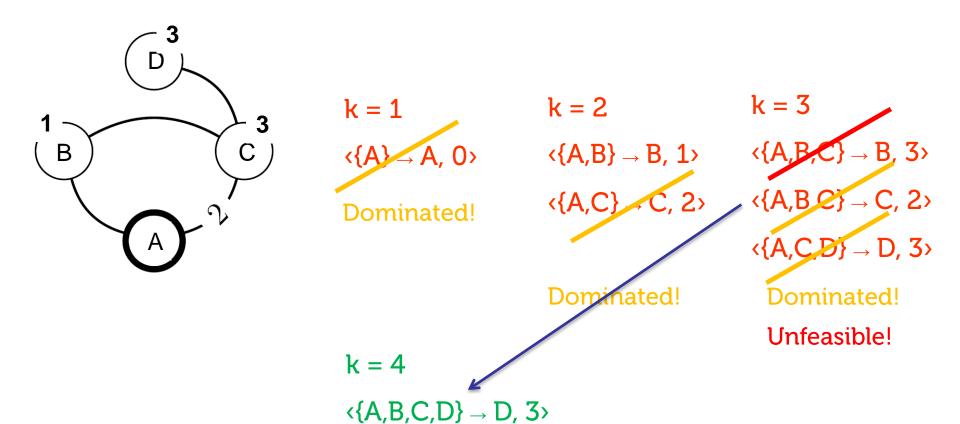














### Computational complexity

The worst-case complexity of the algorithm is

$$O(|T(s)|^2 2^{|T(s)|})$$

since it has to compute proper covering sets up to cardinality |T(s)|.

With annotations of dominances and routes generation, the whole algorithm yields a worst-case complexity of  $O(|T(s)|^52^{|T(s)|})$ .



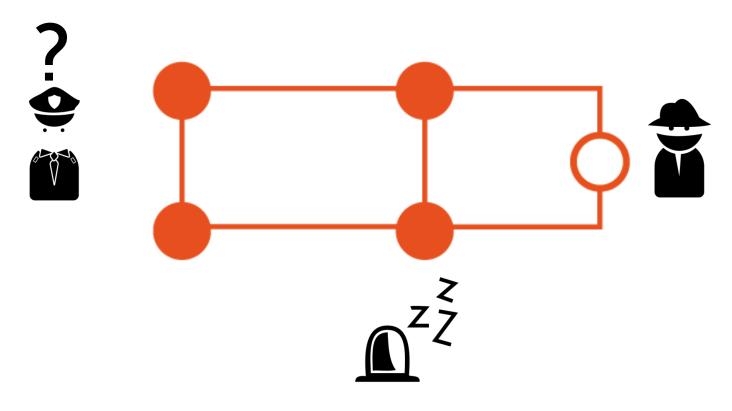
#### Pseudo-code

#### **Algorithm 1** ComputeCovSets (Basic)

```
1: \forall t \in T, k \in \{2, ..., |T|\}, C_t^1 = \{t\}, C_t^k = \emptyset
2: \forall t \in T, c(\lbrace t \rbrace) = \omega_{v,t}^*, c(\emptyset) = \infty
3: for all k \in \{2 ... |T|\} do
4:
          for all t \in T do
               for all Q_t^{k-1} \in C_t^{k-1} do
5:
                   Q^{+} = \{ f \in T \setminus Q_t^{k-1} \mid c(Q_t^{k-1}) + \omega_{t,f}^* \le d(f) \}
6:
7:
                    for all f \in Q^+ do
                        Q_f^k = Q_t^{k-1} \cup \{f\}
 8:
                        U = Search(Q_f^k, C_f^k)
9:
                         if c(U) > c(Q_t^{k-1}) + \omega_{t,f}^* then
10:
                              C_f^k = C_f^k \cup \{Q_f^k\}
11:
                              c(Q_f^k) = c(Q_t^{k-1}) + \omega_{t,f}^*
12:
13:
                         end if
14:
                     end for
15:
               end for
16:
          end for
17: end for
```



#### Missed detections



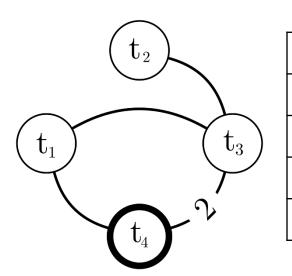
v\* is the best placement u\* is the second best placement

$$(1 - \alpha)(1 - g_{v^*}) > 1 - g_{u^*}$$



#### Missed detections

#### v\* is the best placement for $\alpha \leq 0.25$

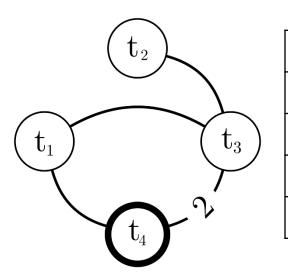


t	π(t)	d(t)	$p(s_1 t)$
t <sub>1</sub>	0.5	1	1.0
t <sub>2</sub>	0.5	3	1.0
<b>t</b> <sub>3</sub>	0.5	2	1.0
t <sub>4</sub>	0.5	2	1.0



#### Missed detections

#### v\* is the best placement for $\alpha \leq 0.50$



t	π(t)	d(t)	p(s <sub>1</sub>  t)
t <sub>1</sub>	1.0	1	1.0
t <sub>2</sub>	1.0	3	1.0
t <sub>3</sub>	1.0	2	1.0
t <sub>4</sub>	1.0	2	1.0



#### k-SRG-v is NP-hard

We reduce from Hamiltonian Path.

Given an instance of HP,  $G_H = (V_H, E_H)$ , we build a k-SRG-v instance as follows:

- V = V<sub>H</sub> U {v};
- $E = E_H U \{(v,h), h \text{ in } V_H\}, w_{i,i} = 1;$
- $T = V_H$ ,  $d(t) = |V_H|$ ,  $\pi(t) = 1$ ;
- $S = \{s\}, p(s|t) = 1;$
- k = 0.

If  $g_v = 0$ , then T must be a covering set that admits at least one covering route r, which visits every node exactly one time.

Since  $T = V_H$ ,  $g_v \le 0$  if and only if  $G_H$  admits a Hamiltonian path.



### Computing covering sets

Definition: The decision problem COV-SET is defined as:

INSTANCE: an instance of SRG-v with a target set T

QUESTION: is T a covering set for some covering route r?

Theorem: COV-SET is NP-complete.



### Security games around the World

