A Multi-Engine Theorem Prover for a Description Logic of Typicality

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Description Logics

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- Important formalisms of knowledge representation
- Two key advantages:
  - well-defined semantics based on first-order logic
  - good trade-off between expressivity and complexity
- at the base of languages for the semantic (e.g. OWL)

Knowledge bases

- Two components:
  - TBox= inclusion relations among concepts
  - ABox= instances of concepts and roles = properties and relations among individuals

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Reasoning

- TBox = taxonomy of concepts
- need of representing prototypical properties and of reasoning about defeasible inheritance
- integration with nonmonotonic reasoning mechanism to handle defeasible inheritance [BH95, BLW06, DLN+98, DNR02, ELST04, Str93]
- all these methods present some difficulties

Our solution

- DLs + typicality operator $\mathbf{T}$ for defeasible reasoning in DLs [GGOP13]
- meaning of $\mathbf{T}$: (for any concept C) $\mathbf{T}(C)$ singles out the "typical" instances of C
- semantics of $\mathbf{T}$ defined by a set of postulates that are a restatement of Kraus-Lehmann-Magidor axioms of preferential logic $\mathbf{P}$

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The logic $\mathcal{ALC} + \mathbf{T}_{min}$

Basic notions

- A KB comprises assertions $\mathbf{T}(C) \sqsubseteq D$
- $\mathbf{T}(Student) \sqsubseteq FacebookUsers$ means "normally, students use Facebook"
- $\mathbf{T}$ is nonmonotonic
  - $C \sqsubseteq D$ does not imply $\mathbf{T}(C) \sqsubseteq \mathbf{T}(D)$

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Example

$$\mathbf{T}(BasketballPlayer) \sqsubseteq \neg Rich$$
$$\mathbf{T}(BasketballPlayer \sqcap NBAMember) \sqsubseteq Rich$$

Reasoning

- ABox:

$$BasketballPlayer(marco)$$

- Expected conclusions:

$$\neg Rich(marco)$$

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- The operator $\mathbf{T}$ is nonmonotonic, but...
- The logic is monotonic
  - If KB $\models F$, then KB' $\models F$ for all KB' $\supseteq$ KB

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- in the KB of the previous slides:
  - if *BasketballPlayer*(*marco*) $\in$ ABox, we are not able to:

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The nonmonotonic logic $\mathcal{ALC} + \mathbf{T}_{min}$ [GGOP13]

Minimal entailment

- Preference relation among models of a KB
  - $\mathcal{M}_1 < \mathcal{M}_2$ if $\mathcal{M}_1$ contains less exceptional (not minimal) elements
  - $\mathcal{M}$ minimal model of KB if there is no $\mathcal{M}'$ model of KB such that $\mathcal{M}' < \mathcal{M}$
- Minimal entailment
  - KB $\models_{min} F$ if F holds in all *minimal* models of KB
- Nonmonotonic logic
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The calculus $\mathcal{TAB}_{min}^{\mathcal{ALC}+\mathbf{T}}$

Basic ideas

- for deciding whether a query F is minimally entailed from a KB
- two-phase computation:
  - Phase 1: verifies whether KB $\cup \{\neg F\}$ is satisfiable building candidate models
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- More precisely: if, for each branch **B** built by Phase 1, either:
  - **B** is closed or
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Design of `DysToPic`

Basic concepts

- multi-engine theorem prover for reasoning in $\mathcal{ALC} + \mathbf{T}_{min}$
- SICStus Prolog implementation of the two-phases tableaux calculus wrapped by a Java interface which relies on the Java RMI APIs for the distribution of the computation
- "worker/employer" paradigm
  - the computational burden for the "employer" can be spread among an arbitrarily high number of "workers" which operate in complete autonomy, so that they can be either deployed on a single machine or on a computer grid

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- no need for Phase 1 to wait for the result of one elaboration of Phase 2 on an open branch, before generating another candidate branch
  - in order to prove whether F entails from a KB, Phase 1 can be executed on a machine
  - every time that a branch remains open after Phase 1, the execution of Phase 2 for this branch is performed in parallel on a different machine
  - meanwhile, the main machine can carry on with the computation of Phase 1
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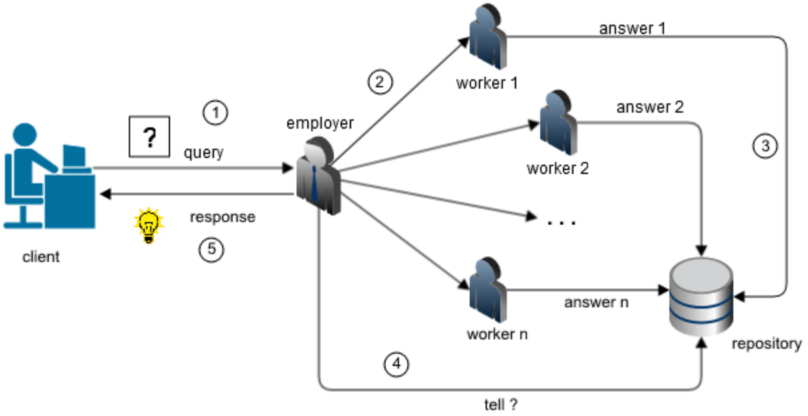
The architecture of `DysToPic`



Technologies

- Tableaux rules implemented in SICStus Prolog
- Library `se.sics.jasper` to combine Java and SICStus Prolog and to decouple Phase 1 and Phase 2
- Concurrency via multithreading and RMI (Java)

Performances

- Comparison with a standard implementation PreDeLo
- Promising performances
  - `DysToPic` is better than the competitor in answering that F is not minimally entailed from KB
  - surprisingly enough, better performances also in case F is minimally entailed from KB
- advantages of distributing the computation justify the overhead of the machinery needed for that

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