Learning Accurate Cutset Networks by Exploiting Decomposability

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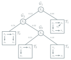
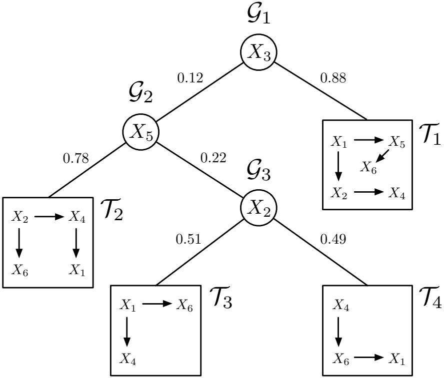
Introduction

Tractable Probabilistic Graphical Models



- Probabilistic Graphical Models
  - powerful formalism to model rich and structured domains
  - capture independences among random variables into a graph
  - computing exact inference in PGMs is a NP-Hard problem
- Tractable Probabilistic Graphical Models
  - provide exact and efficient inference but less expressive
  - tree-structured models, Bayesian and Markov Networks compiled into Arithmetic Circuits, and Sum-Product Networks
- Cutset Networks
  - weighted probabilistic model trees
  - OR-trees having tree-structured models as leaves
  - non-negative weights on inner edges
  - Inner nodes, i.e., conditioning OR nodes, are associated to random variables and outgoing branches represent conditioning on the values for those variables domains.

Cutset Networks



Given $\mathbf{X}$ be a set of discrete variables, a *CNet* is defined as follows:

1. a CLtree, with scope $\mathbf{X}$, is a CNet;
2. given $X_i \in \mathbf{X}$ a variable with $|Val(X_i)| = k$, graphically conditioned in an OR node, a weighted disjunction of k CNets $\mathcal{G}_i$ with same scope $\mathbf{X}_{\setminus i}$ is a CNet, where all weights $w_{i,j}$, $j = 1, \ldots, k$, sum up to one, and $\mathbf{X}_{\setminus i}$ denotes the set $\mathbf{X}$ minus the variable X_i.

Contribution

dCSN

The dCSN algorithm

- avoiding decision tree heuristics
  - choosing the best variable directly maximizing the log-likelihood
- complex structures penalized adopting the BIC

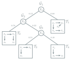
$$\text{score}_{\text{BIC}}(\langle \mathcal{G}, \boldsymbol{\gamma} \rangle) = \log P_{\mathcal{D}}(\langle \mathcal{G}, \boldsymbol{\gamma} \rangle) - \frac{\log M}{2} \text{Dim}(\mathcal{G})$$

- Bagging in order to obtain a mixture of CNets
  - k bootstrapped samples $\mathcal{D}_i$ from the dataset $\mathcal{D}$
    - leading to k CNets $\mathcal{G}_i$
  - resulting bagged CNet $\mathcal{G}$ set to a weighted sum of CNets $\mathcal{G}_i$

$$\hat{P}(\xi : \mathcal{G}) = \sum_{i=1}^{k} \mu_i P(\xi : \mathcal{G}_i),$$

where $\mu_i = \ell_{\mathcal{D}}(\langle \mathcal{G}_i, \boldsymbol{\gamma}_i \rangle) / \sum_{j=1}^{k} \ell_{\mathcal{D}}(\langle \mathcal{G}_j, \boldsymbol{\gamma}_j \rangle)$

Cutset Networks



Proposition 1 (CNet log-likelihood decomposition)

$$\ell_{\mathcal{D}}(\langle \mathcal{G}, \boldsymbol{\gamma} \rangle) = \sum_{\xi \in \mathcal{D}} \sum_{i=1,\ldots,n} \log P(\xi[X_i] | \xi[\mathrm{Pa}_i]) \tag{1}$$

$$\ell_{\mathcal{D}}(\langle \mathcal{G}, \boldsymbol{\gamma} \rangle) = \sum_{j=1,\ldots,k} M_j \log w_{i,j} + \ell_{\mathcal{D}_j}(\langle \mathcal{G}_j, \boldsymbol{\gamma}_{\mathcal{G}_j} \rangle) \tag{2}$$

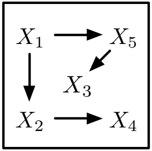
Proposition 2 (BIC decomposition)

$$\ell_{\mathcal{D}_l}(\langle \mathcal{G}_i, \boldsymbol{\gamma}_i \rangle) - \ell_{\mathcal{D}_l}(\langle \mathcal{T}_l, \boldsymbol{\theta}_l \rangle) > \frac{\log M}{2} \tag{3}$$

- instead of recomputing the likelihood on the complete dataset $\mathcal{D}$ we can evaluate only the local improvement
- the decomposition of $\mathcal{T}_l$ is independent from all other $\mathcal{T}_j, j \neq l$ being their local contributions to the global log-likelihood independent
  - it is not significant the order we choose to decompose leaf nodes

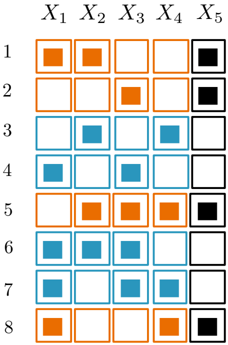
dCSN example I

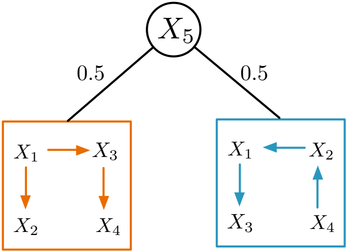
	X_1	X_2	X_3	X_4	X_5
1	■	■			■
2			■		■
3		■		■	
4	■		■		
5		■	■	■	■
6	■	■	■		
7	■		■	■	
8	■			■	■



- starting with a single CLTree for all variables X_1, X_2, X_3, X_4, X_5

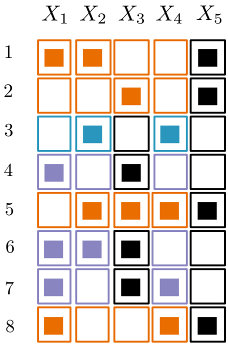
dCSN example II

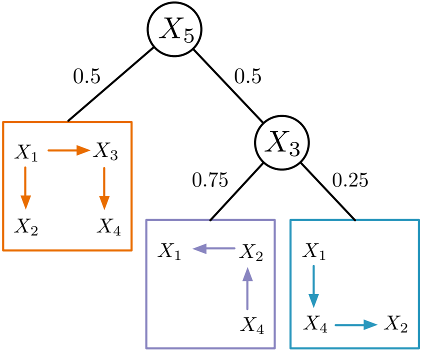




- checking whether there is a decomposition
  - adding OR node on variable X_5 applied on two CLtrees with higher ll

dCSN example III





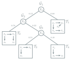
- recursively apply the decomposition process
  - adding OR node on variable X_3 applied on two CLtrees with higher ll

Experiments

Empirical risk for all algorithms

	CNet	CNetP	dCSN	CNet-B	CNetP-B	dCSN-B	MT	MCNet
NLTCS	-6.11	-6.06	**-6.04**	-6.09	-6.02	-6.02	**-6.01**	-6.00
MSNBC	-6.06	**-6.05**	-6.05	-6.06	**-6.04**	-6.04	-6.08	-6.04
Plants	**-13.24**	-13.25	-13.35	-12.30	-12.38	**-12.21**	-12.93	-12.78
Audio	-44.58	**-42.05**	**-42.06**	-42.09	-40.71	**-40.17**	**-40.14**	-39.73
Jester	-61.71	**-55.56**	**-55.30**	-57.76	-53.17	**-52.99**	-53.06	-52.57
Netflix	-65.61	**-58.71**	**-58.57**	-63.08	-57.63	**-56.63**	-56.71	-56.32
Accidents	-30.97	-30.69	**-30.17**	-30.25	-30.28	**-28.99**	-29.69	-29.96
Retail	-11.07	**-10.94**	-11.00	-10.99	-10.88	-10.87	**-10.84**	-10.82
Pumsb-star	-24.65	-24.42	**-23.83**	-24.39	-24.19	**-23.32**	-23.70	-24.18
DNA	-90.48	-87.59	**-87.19**	-90.66	-86.85	**-84.93**	**-85.57**	-85.82
Kosarek	-11.19	**-11.04**	-11.14	-10.97	-10.85	-10.85	**-10.62**	-10.58
MSWeb	-10.07	-10.07	**-9.94**	-9.95	-9.91	-9.86	**-9.82**	-9.79
Book	-37.62	-37.35	**-37.22**	-35.88	-35.62	-35.92	**-34.69**	-33.96
EachMovie	**-59.19**	**-58.37**	**-58.47**	-54.22	-54.02	**-53.91**	-54.51	-51.39
WebKB	-162.85	-162.17	**-161.16**	-156.79	-156.94	**-155.20**	-157.00	-153.22
Reuters-52	**-88.72**	**-88.55**	**-88.60**	-86.22	-86.89	**-85.69**	-86.53	-86.11
BBC	**-262.08**	**-263.08**	**-262.08**	**-252.01**	-257.72	**-251.14**	-259.96	-250.58
Ad	-16.92	-16.92	**-14.81**	-15.94	-16.02	**-13.73**	-16.01	-16.68

Conclusions



- a new approach to learn the structure of CNets model
  - exploiting the decomposable score and maximizing the likelihood
  - formulating a score including the BIC criterion
  - introducing informative priors on smoothing parameters
- mixtures of CNets with bagging as an alternative to EM
- evaluation on standard benchmarks proving the validity of our claims

Future Work

- latent nodes such as in latent tree models
- (gradient) boosting

Code available at `http://www.di.uniba.it/~ndm/dcsn/`